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# Under UtilizedSpectrum Sensing Cognitive Radio Framework for Gaussian Channel

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**Abstract**— In wireless communication systems, spectrum is a very valuable resource and it has been a focal point for research and development efforts over the last several decades. Cognitive radio, which is one of the efforts to utilize the available spectrum more efficiently. Spectrum sensing is a crucial component in the discovery of spectrum opportunities for secondary users (unlicensed users).Cognitive radio framework optimizes secondary throughput while minimizing the interference to primary users (licensed users).Convex optimization under Gaussian channel and Gaussian input signal is considered. Gaussian channel do not account for fading, frequency selectivity, interference , non linearity dispersion.

**Index Terms**—Cognitive radio, MSJD, spectrum sensing, throughput maximization, wideband sensing.

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## 1 Introduction

Spectrum utilization is a critical problem in wireless communication. FCC (Federal communication commission) show that the spectrum utilization in the 0-6 GHz band varies from 15% to 85%.This given birth to cognitive radio. The IEEE has formed a working group (IEEE 802.22) to develop an air interface for opportunistic secondary access to the TV spectrum via cognitive radio technology [1-2]. In CR terminology primary users also called as licensed users and secondary users are called as unlicensed users or cognitive users. The unoccupied frequency band by the primary users called as spectrum holes or white space .The fundamental task of CR network is to detect the licensed users, if they are present then identify the available spectrum. This process is called spectrum sensing.

The objective of spectrum sensing is:

-Secondary users should not cause harmful interference to primary users.

-Secondary users should efficiently identify the spectrum holes for required throughput.

The framework referred for spectrum sensing is Multiband sensing time adaptive joint detection(MSJD).Wide band Gaussian channel is divided into number of non overlapping narrowband sub channels and find the detector parameters individually and then the detector parameters and sensing time are jointly optimized[5].Convex optimization is solved if the practical constraints are imposed[6].This framework maximize the secondary throughtput and minimize the interference of primary user through an adaptive selection of the sensing time and the individual narrow band channel thresholds.

## 2 Channel Sensing

### 2.1 System Model

Consider a wideband channel which is divided into N non overlapping narrowband sub channels and this is shared by J number of primary users. Multicarrier modulation based primary communication system is considered [5].The white space in the $K^{th}$ sub band can be detected by the following hypothesis. $H_{0,k}$ represents the absence of the primary signal and hypothesis

$H_{1,k}$ represents the presence of the primary signal.

$$H_{0,k} : R_k = W_k \quad (1)$$

$$H_{1,k} : R_k = H_k S_k + W_k \quad (2)$$

K point fast fourier transformation of the noise w(n) the discrete time channel impulse response h(n) and primary signal s(n) are given by $W_k$, $H_k$ and $S_k$ respectively. The noise is assumed to be AWGN with mean zero and variance $\sigma_w^2$ .The SNR of the received signal at the $k^{th}$ sub channel given by,

$$\boldsymbol{r_k} = \frac{\boldsymbol{E(|S_k|^2)\,|H_k|^2}}{\boldsymbol{\sigma_\omega^2}} \quad (3)$$

### 2.2 Periodic Sensing

The frame duration T used for periodic sensing consists of one sensing slot of duration τ and one data transmission slot of duration T-τ. Once the secondary user detects an opportunity for transmission it may tune its transmission [8] it may tune its transmission parameters to access the channel. It should continue sensing the spectrum every T seconds in order to vacate the channel if the primary user reappears. For a given sensing time τ the number of samples used for sensing of one sub channel is $M = \tau/T_s$ .where $T_s$ is the sampling time

### 2.3Received Signal

K point discrete fourier transform of the received signal $R_k$ is given by,

$$R_k = \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} r(n)\, e^{-j2\pi nk/N} \quad (4)$$

### 2.4Signal Detection In Each Sub Channel

The decision statistic $T_{k(r)}$ for each sub channel can be written as,

$$T_{k(r)} = \frac{1}{M} \sum_{m=0}^{M-1} |R_k\ (m)|^2, \ \ k = 1,2,\ldots\ldots, N \quad (5)$$

Defining $\varepsilon_k$ as the decision threshold in sub band k the decision statistic can be written as

$Tk(r) > \varepsilon_k$ -- $H_{1,k}$

$Tk(r) < \varepsilon_k$ -- $H_{0,k}$

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which $\varepsilon = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N]^T$ denotes the threshold vector. Probabilities of false alarm and detection for the $k^{th}$ sub channel[11] can be defined as

$$p_f^{(k)}(\varepsilon_{k,\tau}) = p_r(T_k > \varepsilon_k | H_{0,k}) \quad (6)$$

$$p_d^{(k)}(\varepsilon_{k,\tau}) = p_r(T_k > \varepsilon_k | H_{1,k}) \quad (7)$$

In vector this can be expressed as,

$$P_f(\varepsilon,\tau) = [P_f^{(1)}(\varepsilon_k,\tau),\ldots\ldots,P_f^k(\varepsilon,\tau)]^T$$
$$P_m(\varepsilon,\tau) = [P_m^{(1)}(\varepsilon_1,\tau),\ldots\ldots,P_m^{(k)}(\varepsilon,\tau)]^T$$

$$P_m(\varepsilon,\tau) = 1 - P_d(\varepsilon,\tau)$$

Where 1 denotes the all ones vector. The threshold $\varepsilon_k$ is a trade off between $P_f$ and $P_d$. A typical performance metric for a sophisticated [5] detection mechanism is that $P_d$ should be as great as possible whereas $P_f$ as little as possible.

## 3 MSJD Frame Work

We find the detection threshold $\{\boldsymbol{\varepsilon_k}\}_{k=1}^{K}$ and the sensing time $\tau$ so as to optimize the performance of the secondary network while protecting the primary [13] network at its desired level.

### 3.1 Formulation

The percentage of spectrum vacancies detected by the secondary users is $1 - P_f^{(k)}$ and the portion of the frame duration available for oppurtunistis [12] transmission is T-τ/T. Hence the available throughput is

$$R_{(\varepsilon,\tau)} = \left(\frac{T-\tau}{T}\right) o^T(1 - P_f(\varepsilon,\tau)) \quad (8)$$

Where 1 denotes the all-ones vector. $o_k$ denotes the opportunistic throughput of secondary user.

In particular, we define $c_k$ as the cost of interfering with a primary user in the $k$th subchannel and $c = [c_1, c_2, \ldots, c_N]$. Given the fact that primary users may demand different levels of protection, the aggregate interference to primary user $j$ is defined as

$$I_j(\varepsilon,\tau) = \sum_{i\epsilon Sj} C_i P_m^{(i)}(\epsilon_i,\tau). \quad (9)$$

### 3.2 Optimization problem

Mathematically, the optimization problem can be stated as $min_{\varepsilon,\tau}.\ R(\varepsilon,\tau)$

$$I_j(\varepsilon,\tau) \le \xi_j, j = 1,\ldots\ldots J$$

$$P_m(\varepsilon,\tau) \le \alpha$$

$$P_f(\varepsilon,\tau) \le \beta$$

$$\tau \le \tau_{max}$$

Where $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N]$ and $\beta = [\beta_1, \beta_2, \ldots, \beta_N]$ and are the minimum requirements of each subchannel and $\boldsymbol{\xi_j}$ represents the maximum aggregate interference tolerated by the jth primary user. More specifically, $\alpha_k$ shows the interference margin required in the $k$th subchannel and $\beta_k$ forces a minimum detection of frequency holes. $\tau_{max}$ is the maximum limit allowable sensing time.

### 3.3 Minimum Sensing Time

Minimum sensing time min is calculated as $\tau_{min}$=arg $\tau_{min}$
This can be solved by using the following algorithm

### 3.4 Convex Optimization

In order to make the analysis easier, we reformulate the problem as

$$\min_{\varepsilon,\tau}.\ R_{miss}(\varepsilon,\tau)$$

$$R_{miss}(\varepsilon,\tau) = r^T[P_f(\varepsilon,\tau)\left(1 - \frac{\tau}{T}\right) + 1\frac{\tau}{T}] \quad (10)$$

## 4 Convex Analysis Of MSJD Framework

Under certain condition the convex optimization is addressed for sensing time and detection threshold

(A)For a large number of samples the pdf of $T_{k(r)}$ under H0,k and $H_{1,k}$ approximated to Gaussian distribution according to central limit theorem.

$$T_k(r) \sim \begin{matrix} N(\mu_{0,k}, \sigma_{0,k}^2) underH_{0,k} \\ N(\mu_{1,k}, \sigma_{0,k}^2) underH_{1,k} \end{matrix}$$

Wher $\mu_{0,k}$ ,$\mu_{1,k}$ , and $\sigma^2_{o,k}$, $\sigma^2_{1,k}$, represents mean and variance for hypothesis $H_{0,k}$ and $H_{1,k}$ .respectively.The decision rules for the chosen scenarios are

The primary signal is BPSK modulated and noise is real Gaussian

$$T_k(r) \sim \begin{cases} N(\sigma_w^2, 2\sigma_w^4 M^{-1}) underH_{0,k} \\ N\sigma_w^2(\gamma_k + 1), 2\sigma_w^4(\gamma_k + 1)^2 M^{-1} underH_{1,k} \end{cases}$$

(B) The probability of false alarm given by

$$P_f^{(k)}(\varepsilon_k,\tau) = Q\left(\frac{\varepsilon_k - \mu_{0,k}}{\sqrt{\sigma_{0.k}^2}}\right)$$

According the chosen scenario false alarm given by

$$P_{fa}^{(k)}(\varepsilon_k,\tau) = P_{fb}^{(k)}(\varepsilon_k,\tau) = Q\left(\left(\frac{\varepsilon_k}{\sigma_w^2} - 1\right)\sqrt{\frac{\tau f_s}{2}}\right)$$

(C) The probability of detection given by

$$P_d^{(k)}(\varepsilon_k,\tau) = Q\left(\frac{\varepsilon_k - \mu_{1,k}}{\sigma_{1,k}^2}\right)$$

The convexity range for probability of detection and false alarm are derived.

Observation: 1 The function $P_{fa(k)}$ is convex in $\varepsilon_k$ and $\tau$ if

$$P_{fa}^{(k)}(\varepsilon_k,\tau) = P_{fb}^{(k)}(\varepsilon_k,\tau) \le \mathrm{Q}\left(\frac{1}{\sqrt{3}}\right)$$

The hessian function can be calculated as $C_k \times$

$$\begin{bmatrix} \left(\frac{\varepsilon_k}{\sigma_w^2}-1\right)\tau^2 f_s \sigma_w^{-2}\left(\frac{\varepsilon k}{\sigma_w^2}-1\right)^2 \frac{\tau f_s}{2}-1 \\ \left(\frac{\varepsilon_k}{\sigma_w^2}-1\right)^2 \frac{\tau f_s}{2}-1\left(\frac{\varepsilon_k}{\sigma_w^2}-1\right)\frac{\tau^{-1}\sigma_w^2}{2}+\left(\frac{\varepsilon_k}{\sigma_w^2}-1\right)^3 \frac{f_s\sigma_w^2}{4} \end{bmatrix}$$

$$Det(.) = C_k^2 \times \left(\frac{3}{2}\left(\frac{\varepsilon_k}{\sigma_w^2}-1\right)^2 \tau f_s - 1\right)$$

The determinant is non negative if

$$\left(\frac{\varepsilon_k}{\sigma_w^2}-1\right)\sqrt{\frac{\tau f_s}{2}} \ge \sqrt{\frac{1}{3}}$$

Observation:2 The function $p^{(k)}_{ma}$ is convex in $\varepsilon_k$ and $\tau$ if

$$P_{ma}^{(k)}(\varepsilon_k,\tau) \le \mathrm{Q}\left(\frac{1}{\sqrt{3}}\right)$$

The hessian function can be calculated as

$$C_k \times$$

$$\begin{bmatrix} \left(\frac{\tau^2 f_s}{(2\gamma_k+1)\sigma_w^2}\right)\left(\frac{\varepsilon k}{\sigma_w^2}-\gamma_k-1\right)\frac{\tau f_s}{2(2\gamma_k+1)}\left(\frac{\varepsilon_k}{\sigma_w^2}-\gamma_k-1\right)^2-\frac{1}{2} \\ \frac{\tau f_s}{2(2\gamma_k+1)}\left(\frac{\varepsilon_k}{\sigma_w^2}-\gamma_k-1\right)^2-\frac{1}{2}\frac{\left(\frac{\varepsilon_k}{\sigma_w^2}-\gamma_k-1\right)^3 f_s}{4\sigma_w^{-2}} \end{bmatrix}$$

where

$$C_k = \frac{1}{4\sigma_w^2\sqrt{\pi}}\sqrt{\frac{f_s}{\tau(2\gamma_k+1)}}\exp\left[-\left(\frac{\varepsilon_k}{\sigma_w^2}-\gamma_k-1\right)^2\frac{\tau f_s}{4(2\gamma_k+1)}\right]$$

$$Det(.) = \frac{\tau f_s}{(2\gamma_k+1)}\frac{3}{2}\left(\frac{\varepsilon_k}{\sigma_w^2}-\gamma_k-1\right)^2-1$$

This is non negative if

$$\sqrt{\frac{\tau f_s}{2(2\gamma_k+1)}}\left(\frac{\varepsilon_k}{\sigma_w^2}-\gamma_k-1\right) \ge \frac{1}{\sqrt{3}}$$

Observation:3 The function $p^{(k)}_{mb}$ is convex in $\varepsilon_k$ and $\tau$ if

$$P_{ma}^{(k)}(\varepsilon_k,\tau) \le \mathrm{Q}\left(\frac{1}{\sqrt{3}}\right)$$

The hessian function can be calculated as

$$C_k \times$$

$$\begin{bmatrix} \left(\frac{\tau^2 f_s}{(\gamma_k+1)\sigma_w^2}\right)\frac{\tau f_s}{2}E_k - E_k^{-1} \\ \frac{\tau f_s}{2} - E_k - E_k^{-1}\frac{(\gamma_k+1\sigma_w^2)}{2\tau}+\frac{(\gamma_k+1)\sigma_w^2 f}{4}E_k^2 \end{bmatrix}$$

$$Det(.) = C_k^2 \times \left(\frac{\tau f_s}{(2\gamma_k+1)}\frac{3}{2}\left(\frac{\varepsilon_k}{\sigma_w^2}-\gamma_k-1\right)^2-1\right)$$

This is non negative if

$$\sqrt{\frac{\tau f_s}{2(2\gamma_k+1)}}\left(\frac{\varepsilon_k}{(\gamma_k+1)\sigma_w^2}-1\right) \ge \frac{1}{\sqrt{3}}$$

Observation:4 The function $R_{loss}$ is convex in $\varepsilon_k$ and $\tau$ if

$$P_f^{(k)}(\varepsilon_k,\tau) \le Q\left(\frac{1}{\sqrt{3}}\right) and \frac{\tau}{T} \le 0.5$$

Both the objective and constraint are convex

$$0 \le \alpha_k \le Q\frac{1}{\sqrt{3}} \qquad k = 1,2.......K$$

$$0 \le \beta_k \le Q\frac{1}{\sqrt{3}} \qquad k = 1,2.......K$$

$$0 \le \tau_{\max} \le 0.5\, T$$

## 5 SIMULATION

In this section computer simulation results are presented to evaluate the proposed MSJD framework for complex gaussian channel conditions.A single PU communication is consider with a wideband spectrum of 48MHz and it is divided into 8 subchannels.Each subchannel is characterized by different parameters in which $O_k$ , $\gamma_k$, and $c_k$ mention throughput in Mbps the received SNR and cost coefficient respectively.Moreover we assume the probability of detection of primary signal for each subchannel to be 80%($\alpha_k$ =0.2) and opportunistic detection margin for all subchannel is assumed to be

50%( $\beta_k$ =0.5).

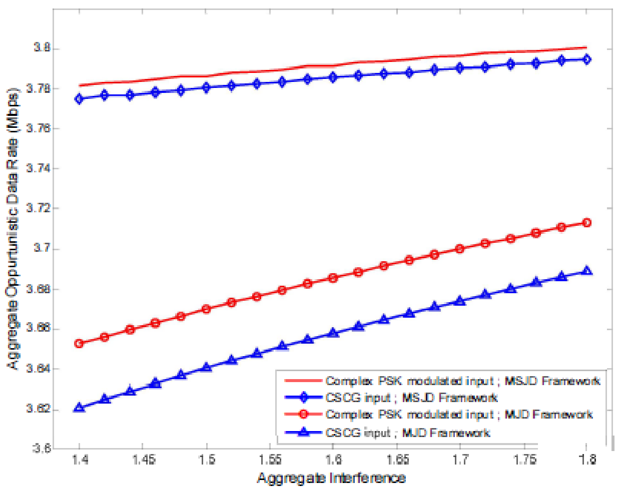


Fig:1 Available secondary throughput vs aggregate interference

Fig :1 shows the comparative performance of MSJD and MJD frameworks for Gaussian channel.We infer from the figure that dynamically selecting the sensing slot duration is more advantageous than MJD also it depicts the superiority of employing a complex PSK modulated input signal.

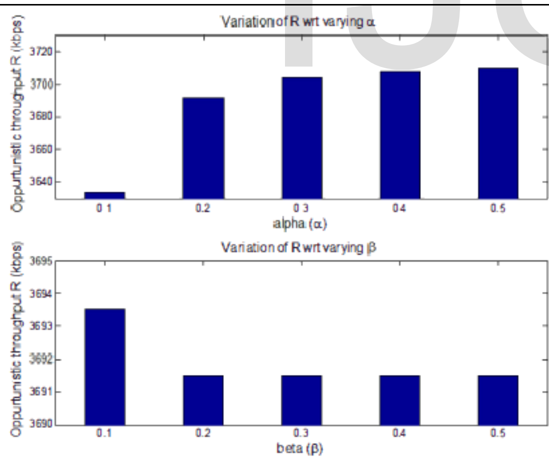


Fig:2 Oppurtunistic throughput versus increment in $\alpha$ and $\beta$

Fig:2 Represents the opportunistic throughput versus probability of interference with PU $\alpha$ and detection margin β.For a given value of β=0.5 and c=1.4 the secondary throughput increases at a cost of increased interference to the primary network but the aggregate interference to the PU remains constant.When $\alpha$ =0.1 the variations of β slightly decreases the output.

## 6 Conclusion

For real Gaussian channel multiband sensing time adaptive joint detection technique is investigated and the underutilized. spectrum is sensed and it is adapted by the secondary users. Primary user interference is below the desired level. By using this technique underutilized spectrum is sensed and in the adaptive manner and used for new application. Due to this method the spectrum scarcity problem is overcome .

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